UNCLASSIFIED

AD NUMBER
AD323742
CLASSIFICATION CHANGES
TO: **unclassified**
FROM: **confidential**
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited.**
FROM: **Controlling Organization: British Embassy, 3100 Massachusetts Avenue, NW, Washington, DC 20008.**
AUTHORITY
DSTL, AVIA 6/21487, 11 Dec 2008; DSTL, AVIA 6/21487, 11 Dec 2008

THIS PAGE IS UNCLASSIFIED

TECH. NOTE
G.W. 564

CONFIDENTIAL

TECH. NOTE
G.W. 564

ROYAL AIRCRAFT ESTABLISHMENT
(FARNBOROUGH)

TECHNICAL NOTE No: G.W. 564

AN ASSESSMENT OF THE ACCURACY OF MEASUREMENT OF GYRO WANDER RATE ON THE BLACK KNIGHT STABILIZED PLATFORM UNDER PRE-LAUNCH CONDITIONS

by

G. E. BELTRAME, B.E.

DECEMBER, 1960

MINISTRY OF AVIATION

CONFIDENTIAL

U.D.C. No. 531.383:53.082.16:53.082.74:623.451-519(Black Knight)

Technical Note No. G.W.564

December, 1960

AD-323742

ROYAL AIRCRAFT ESTABLISHMENT

(FARNBOROUGH)

AN ASSESSMENT OF THE ACCURACY OF MEASUREMENT OF GYRO WANDER RATE ON THE BLACK KNIGHT STABILIZED PLATFORM UNDER PRE-LAUNCH CONDITIONS

by

G. E. Beltrame, B.E.

OTIA10736

SUMMARY

This Note gives an assessment of the accuracy with which the gyro wander rates, due to mass unbalance and ligament torques, can be measured with the Black Knight stabilized platform on the launcher. The gyro wander rates are determined from measurements of gyro torque motor current which are taken with the platform servoed to a fixed set of earth axes. The main sources of error are examined in detail and their overall effect on the accuracy of the wander rate measurements is derived. A further indication of the accuracy possible on the launcher has been obtained by taking a number of measurements on a stabilized platform in the laboratory.

The theoretical assessment of accuracy showed that gyro wander rate could be measured with a standard deviation of uncertainty of better than 0·03 deg/hr. Measurements taken in the laboratory gave a similar value of uncertainty.

LIST OF CONTENTS

LIST OF TABLES

LIST OF TABLES (CONT'D)

LIST OF ILLUSTRATIONS

1 INTRODUCTION

The Black Knight stabilized platform, which is to be used for testing inertial navigation components under flight conditions, contains three single axis integrating gyros which ideally provide a space stabilized reference frame for the accelerometers. Due to unbalance and ligament torques acting about the gyro output axes, the reference frame is not fixed relative to a set of space axes.

One of the pre-launch checks to be performed on the platform consists of measuring the wander rate of the gyros in space due to the above torques. These tests, which give the wander rates of the gyros in a 1g gravitational field will give an indication of the performance of the stabilized platform under flight conditions.

This paper gives an assessment of the accuracy with which the gyro wander rates can be measured while the platform is on the launcher. The estimation of gyro wander rate from torque motor current is subject to two types of error,

(a) the orientation of the gyros is not known exactly,

(b) the instruments recording the torque motor current have a limited accuracy.

In the laboratory, a stabilized platform was servoed to a fixed set of earth axes and the wander rate of each gyro was determined from measurements of torque motor current. The wander rate of each gyro was then obtained by a more accurate method viz. the platform was stabilized solely by the gyros and the wander rate of the platform was measured relative to a pair of autocollimators which were space stabilized to a relatively high degree of accuracy. A comparison was then made of the wander rates obtained by the two methods.

2 CALIBRATION TECHNIQUE

2.1 Relation between gyro wander rate and torque motor current

A description of a single axis integrating gyro is given in Ref.1. It consists essentially of a gyro wheel which has freedom of rotation about one other axis, (called the output axis (O.A.)), besides the spin axis. Attached to the output axis are (a) an angular pick-off for detecting rotation and (b) a torque motor providing torque about the output axis porportional to current. The gyro wheel, enclosed in a can, is surrounded by a viscous fluid, which makes the can neutrally buoyant and also provides a restraining torque about the output axis proportional to angular velocity. The direction of the spin axis, when the pick-off output is zero, is defined as the spin reference axis (S.R.A.), and the axis which is normal to the plane containing the spin reference axis and output axis is called the input axis (I.A.) of the gyro. The general equation of motion about the gyro output axis is given by

$$H\omega = J(\ddot{\phi} + \ddot{\psi}) + \eta\dot{\phi} + K_T I + M_E$$

where $H\omega$ = gyroscopic torque about the output axis

H = angular momentum of the wheel about its spin axis

ω = space rate of rotation of the gyro about its input axis

J = inertia of wheel and can about the output axis.

$\dot{\phi}, \ddot{\phi}$ = angular velocity and acceleration, respectively, of the wheel and can about the output axis, relative to the gyro casing.

$\ddot{\psi}$ = angular acceleration of the gyro case about the output axis, relative to space axes

η = viscosity of the fluid surrounding the can

K_T = current-torque sensitivity of the torque motor

I = torque motor current

M_E = error or unbalance torques acting about the output axis.

When the gyro is used on a stabilized platform the terms $\dot{\phi}$, $\ddot{\phi}$, $\ddot{\psi}$ will be zero if no angular accelerations are applied to the platform.

Equation 2.1 then reduces to

$$H\omega = K_T I + M_E$$

rearranging terms and dividing by H

$$\frac{M_E}{H} = \omega - \left(\frac{K_T}{H}\right) I \qquad 2.2$$

The term $\frac{M_E}{H}$ is called the gyro wander rate and is, in effect, the input angular rate necessary to overcome the error torques. The term $\frac{K_T}{H}$ is the torque motor current-wander rate sensitivity; the method of obtaining this calibration is given in Ref.2.

The wander rate of a gyro is normally divided into two main components,

(a) a constant wander rate which is caused by power leads etc. applying fixed torques about the output axis,

(b) mass unbalance wander rate, caused by static unbalance of the gyro wheel and can, about the output axis. This wander rate is proportional to the magnitude and direction of the gravitational force acting on the gyro. The mass unbalance moment arm will normally have a component along the spin reference axis (M.U.S.R.A.) and along the input axis (M.U.I.A.)

2.2 Method of calibration on launcher

Three Kearfott single axis integrating gyros are used to control the motion of the platform (see Fig.1), the platform being caused to rotate until the pick-off output (ϕ) is zero. While the platform is on the launcher it is servoed to a fixed set of earth axes by means of optical alignment loops on the roll and pitch axes and an accelerometer levelling loop on the yaw axis (see Ref.3). The error signals from these loops provide the torque motor currents necessary to (a) precess the gyros and hence the platform at earth's rate and (b) overcome the unbalance torques acting on the gyros. Since the earth's rate component about the input axis of each gyro (ω) can

be determined from the platform orientation, then the wander rate of the gyro can be obtained from equation 2.2, provided the torque motor current is measured.

In order to separate the constant wander from mass unbalance wander rate, it is necessary to measure torque motor current for a number of orientations of the platform. Provision has been made for orientating the platform in four positions at 90 deg about the platform pitch axis. This is achieved by servoing the platform to the optical loops through each of the four platform alignment mirrors (see Fig.1) in turn. The equations relating torque motor current and wander rate for each of these positions is given in Appendix 1.

3 SOURCES OF ERROR IN DETERMINING GYRO WANDER RATE

The estimation of gyro wander rate from records of torque motor current, is subject to two types of error:-

(a) The orientation of the gyro input axis is not known exactly.

(b) The instruments recording torque motor current have a limited accuracy.

The sources of error will now be discussed in detail.

3.1 Misalignment between gyro input axis notch and datum surface

The Kearfott gyro (see Fig.2) is supplied with a mounting flange and an accurately machined locating notch. The plane of the mounting flange is ideally normal to the gyro output axis and the position of the notch defines the direction of the input axis.

The reference plane, which is used to locate the direction of the gyro input axis, is the plane which is normal to one of the four platform alignment mirrors and which ideally contains the roll axis of the platform. The mirror which provides this reference is called the datum mirror and its surface is flat to within a few wavelengths of light.

Due to tolerances allowed in the machining of the roll ring and gyro mounting bracket, the gyro mounting flange and hence the gyro input axis may be misaligned from the reference plane and datum mirror. The misalignment of the input axis appears as a rotation about either the gyro output axis or about the spin reference axis. The specification for the machining and assembly of the platform allows a tolerance of ±5 arcmin. in positioning the gyro input axis notch about both the directions described above.

3.2 Gyro manufacturing tolerances

The plane of the mounting flange and the locating notch define the directions of the input, output and spin reference axes of the gyro. To allow for errors in assembly, the gyro performance specification gives a tolerance of ±3 arcmin in positioning the input axis notch on the mounting flange, and the plane of the mounting flange should be normal to the output axis to within ±3 arcmin.

3.3 Misalignment of secondary mirrors

Three mirrors besides the datum mirror are used for locating the platform at 90 deg intervals about the pitch axis. The platform assembly specification states that the normal to these mirrors should lie within ±1 arcmin of the reference plane, and that they should locate the platform at 90 deg intervals to within ±1 arcmin.

Because the normal to the platform alignment mirrors is at 14° 48' to the yaw axis, a rotation θ about the yaw axis will cause a rotation θ tan 14° 48' about the roll axis. This will be called a mirror cross coupling effect.

3.4 Accelerometer alignment errors

A Kearfott single axis accelerometer, with its input axis along the direction of the platform pitch axis, locates the platform about the yaw axis by causing it to rotate until the accelerometer output is zero. The normal to the plane on which the accelerometer feet rest defines the direction of its nominal input axis and, ideally, it should be normal to the reference plane of the platform. The tolerance allowed in fixing this axis normal to the reference plane is ±2 arcmin.

3.5 Accelerometer axis and bias errors

Axis error (see Ref.4) occurs when the normal to the plane containing the accelerometer pivot and centre of gravity of the pendulum (called the true input axis) is not parallel to the nominal input axis. The error may appear as a rotation about both the pivot and an axis passing through the centre line of the pendulum.

Bias error occurs when the null position of the mechanical spring does not coincide with the null position of the electrical spring of the accelerometer.

As a result of these errors, the steady state position of the nominal input axis will not remain horizontal, thus causing the gyros to be misaligned from their assumed orientation. Axis error reverses sign as the platform is turned through 180 deg and the net effect of the two reversals is to maintain the platform in one plane. Bias error is independent of platform position and when the platform is reversed a small change in the platform attitude will occur.

The performance specification for the accelerometer allows a bias error of ±0.0008g maximum, which means that the steady state position of the nominal input axis may vary from the horizontal by ±2·75 arcmin. The maximum axis error allowed is ±0·0002g about either the pivot axis or about the centre line of the pendulum. This implies a maximum error of ±40 arcsec in locating the nominal input axis of the levelling accelerometer to a horizontal plane. For other purposes in the trial, means existed for establishing the position of the true input axis of the accelerometer relative to the datum mirror to ±30 arcsec and bias errors were known to ±0·0001g. Hence, had it proved necessary, a large portion of the accelerometer errors could have been removed. Table 3 shows that little improvement in accuracy would be obtained if this were done.

3.6 Optical alignment and accelerometer levelling loop errors

The optical alignment and accelerometer levelling loops (see Ref.3) locate the platform alignment mirror and the true input axis of the accelerometer to a fixed set of axes defined by the direction of autocollimator line of sight and the local vertical. The optical alignment loops are subject to drift and velocity lag errors and the autocollimator line of sight direction is subject to survey errors. It is estimated that the platform alignment mirror can be aligned to a known direction to within ±10 arcsec. The levelling loop is also subject to drift and velocity lag errors and under the worst conditions a maximum error of ±30 arcsec is expected from this loop.

3.7 Recorder uncertainty

The gyro torque motor current will be measured by recording the voltage across a precision readout resistor which is in series with the torque motor. The recorder to be used is a continuous trace, electronic strip chart recorder, which has a maximum error of ±0·25 per cent of full scale. The width of the recording chart is approximately ten inches. Full scale deflection will be made equivalent to 5 mA of torque motor current or 15 deg/hr of earth's rate for all gyros. The maximum error in any reading will then be ±0·038 deg/hr.

It will be seen from Appendix 1 that the torque motor current in the roll and yaw gyros reverses when the platform is turned through 180 deg. In order to achieve the maximum possible sensitivity, the roll and yaw inputs to the recorder will both be reversed when torque motor current is reversed.

The main sources of error in a recorder of the type used, are

(a) Variation of the emf of the standard cell.

(b) Variation of resistivity of the slidewire along its length.

(c) Backlash and friction in the pen drive.

(d) Uneven spacing of the lines on the recording chart.

(e) Uncertainty in measuring the deflection of the recorded trace.

If, during the course of the test, an accurate calibration is made of recorder pen deflection per unit gyro torque motor current, the source of error (a) will be virtually eliminated. This calibration would involve the use of a vernier potentiometer with an accuracy of an order better than that of the recorder. The error (d) can be eliminated by using an accurate scale to measure deflection, instead of relying on the line spacing.

The performance specification, for the recorder used, gives the sensitivity as better than 0·1 per cent of full scale, which means that backlash and friction errors are of this order.

3.8 Errors in torque motor scale factor

A description of the method used for calibrating the gyro torque motor is given in Ref.2. Basically, the method consists of applying a known current to the torque motor and measuring the wander rate about the gyro input axis which keeps the pick-off at the null position. This gives a scale factor of $\frac{\text{wander rate}}{\text{unit current}}$.

The main sources of error in this calibration are:-

(a) Variation in gyro wheelspeed.

(b) Error in measuring the angle swept out by the gyro input axis.

(c) Uncertainty in measuring the time taken for the gyro to wander through a given angle.

(d) Uncertainty in measuring the torque motor current.

In addition, the torque motor will be subject to non-linearity and the scale factor will vary from day to day. As far as tests on the launcher are

concerned, the main source of error will be the variation of scale factor from day to day. Test results in Ref.2 show a variation of scale factor of 1 part in 600 over twelve days. This figure will be used in assessing the overall accuracy on the launcher.

4 ASSESSMENT OF OVERALL UNCERTAINTY IN GYRO WANDER RATE

4.1 Type of error distribution curve assumed

The errors arising from the sources described in section 3 produce an uncertainty in the recorded value of gyro wander rate. In assessing the overall uncertainty, it has been assumed that these errors possess two properties:-

(a) All the errors are accidental, i.e. they vary in a random manner on different platforms.

(b) All the errors vary independently of each other.

Assumption (a) makes it possible to assume a normal distribution curve for the magnitude of any one error appearing on any platform. The errors which are least likely to follow this pattern, are the platform machining errors - mainly because of lack of samples. However, it has been found possible to keep the machining errors well inside the limits given in section 3, so that the assumed distribution curve is possibly pessimistic.

To obtain the standard deviation σ, of each error, it was assumed that the probability of the errors in section 3 remaining inside the tolerances specified, was 90 per cent i.e. the tolerances were equivalent to a $\sqrt{3}\sigma$ deviation approximately. This assumption was made to allow for lack of numbers of samples.

If the standard deviation of the individual errors is known, then, since all the errors vary independently of each other, the standard deviation of the overall error is given by (see Ref.5)

$$\sigma_{\text{overall}} = \sqrt{\Sigma\,\sigma^2_{\text{individual errors}}}$$

4.2 Method of assessing overall error

The relation between gyro torque motor current and the current required to overcome mass unbalance and ligament torques is given in Appendix 1, for all orientations of the platform. In general, the mass unbalance and constant wander terms appear together, but, whereas the ligament torque acts in a fixed direction about the output axis, the mass unbalance torque reverses direction if the direction of the gravity vector is reversed. Hence, the mass unbalance and constant wander rates may be separated by taking two torque motor current measurements with the platform rotated through 180 deg about a horizontal axis. The appropriate equations for separating mass unbalance and constant wander terms are given in Table 1. It will be seen that in both the yaw and pitch gyros it is possible to obtain only one component of the mass unbalance - the other component could be obtained by rotating the platform about a horizontal axis at 90 deg to the pitch axis. This would have entailed major re-design of the platform and was not warranted because the unknown components of mass unbalance would be in a direction almost parallel to the thrust vector acting on the missile and would have little effect on the wander rate of the platform.

The errors in sections 3.1 to 3.6 cause a misalignment of the gyro input axis. The resultant error in terms of deg/hr of gyro input rate is obtained as follows,

let OXYZ (Fig.3) represent a set of reference axes (in this case the missile roll yaw and pitch axes)

let $\omega_x, \omega_y, \omega_z$ = earth's rate components along the X, Y, Z axes respectively

θ = misalignment of the gyro input axis from the OZ axis, in the OZY plane.

The assumed value of earth's rate detected by the gyro is equal to ω_z.

Actual component $= \omega_z \cos\theta + \omega_y \sin\theta$

but since θ is small

actual component $= \omega_z + \omega_y \theta$

hence the error $= \omega_y \theta$

The sources of error are listed with their standard deviation in Table 2. The error source 3·6 is relatively small and will be neglected. Also, since all the errors produce small misalignments (θ) they may be treated independently of one another.

Although all the errors vary at random from one platform to the other, the errors appearing on one particular platform may be divided into two types.

(a) Errors which remain constant in magnitude independent of platform orientation viz. 3.1, 3.2, 3.4, 3.5, 3.8.

(b) Errors which vary at random as the platform orientation changes viz. 3.3, 3.7.

A brief note is necessary about error source 3.8. Although the error arising from 3·8 varies with the amount of earth's rate detected by the gyro, the effect of rotating the platform through 180 deg. (plus error angle) does not change the earth's rate component by more than 1 per cent. Hence for the present purposes, the error 3.8 may be regarded as constant for one particular pair of current measurements.

The method of adding the individual errors when two readings of torque motor current are added or subtracted is given below.

Let $I_\alpha, I_{(180+\alpha)}$ = torque motor current at the orientations α, $(180+\alpha)$ respectively.

$I^E_\alpha, I^E_{(180+\alpha)}$ = error current produced by one source when the platform is in the α, $(180+\alpha)$ positions respectively.

Taking as an example the case of the roll gyro

$$I_{CW} = \frac{|I_\alpha| - |I_{(180+\alpha)}|}{2} \quad \text{nominally (Table 1)}$$

If the error remains constant in magnitude it is added algebraically

$$\text{i.e. } I_{CW}\ \text{(error)} = \frac{\left|I^E_\alpha\right| - \left|I^E_{(180+\alpha)}\right|}{2} \simeq 0$$

The effect of bias error 3.5b provides an exception i.e. when all the other errors of type (a) above cancel out, bias error does not, and vice versa. This is because all the other error sources change the sign of the error current when the torque motor current reverses whereas bias error does not.

If the error varies randomly as in case (b) above, the resultant error is obtained by statistical addition

$$\text{i.e. } I_{CW}\ \text{(error)} = \frac{\sqrt{\left[I^E_\alpha\right]^2 + \left[I^E_{(180+\alpha)}\right]^2}}{2}$$

A similar treatment applies to the mass unbalance error. These equations have been used in Table 3 to assess the uncertainty in the mass unbalance and constant wander rates of the roll gyro for the 0 deg and 180 deg positions. The constant wander and mass unbalance wander rate uncertainty for all the platform gyros is given in Table 4.

5 EXPERIMENTAL CHECK ON ACCURACY OBTAINABLE

5.1 Method of calibration

In order to check on the accuracy with which gyro wander rates could be measured on the launcher, a number of tests were performed on an experimental platform stabilized by three Kearfott integrating gyros (see Fig.4). The testing was carried out in two stages, -

(a) the platform was aligned to a known set of earth axes by means of optical alignment and accelerometer levelling loops, while the gyro torque motor current was being recorded,

(b) the optical and accelerometer loops were broken so as to allow the gyros to space stabilize the platform. The actual wander rate of each gyro was then determined by observing the wander rate of the platform through an autocollimator whose line of sight was fixed in space i.e. it was space stabilized. The gyro wander rate obtained from this measurement served as a reference for comparing the wander rate derived from torque motor current.

Because of the construction of the stabilized platform, it was not possible to take measurements of torque motor current in two positions at 180 deg. All measurements of current were therefore taken with the platform in one position only. The earth's rate component of the current in each gyro, was determined from the orientation of the platform, the latter being obtained by autocollimating with a theodolite onto two mirrors which were attached to the platform at right angles to each other.

The optical alignment and accelerometer levelling loops were similar to the ones which will be used on the Black Knight launcher. Torque motor current was recorded on a precision recorder with characteristics similar to those given in section 3.7.

Two autocollimators set up at right angles, measured the wander rate about the platform axes. The autocollimators were space stabilized by the

equatorial mount on which they rested (see Fig.4). The equatorial mount consists of a cradle (on which the autocollimators are clamped) which is supported by a shaft whose axis is parallel to the earth's polar axis. Space stabilization is obtained by rotating the shaft with an angular rate equal and opposite to earth's rate, by means of a synchronous motor.

5.2 Experimental results

It was found that the record of torque motor current contained comparatively large components of both high and low frequency noise. The high frequency noise which was of the order of 10 to 20 cycles per sec, was associated with platform disturbances, whereas the low frequency noise appeared at the natural frequency of the optical and accelerometer loops which was of the order of $^1/_{10}$ cycle per sec or less. The high frequency noise was filtered out by an R-C filter with a time constant of 5 sec and the mean value of the record was obtained by integrating the area under the trace, over at least five periods of the low frequency noise, with a planimeter. The peak to peak value of the low frequency noise was, in general, well under 1 deg/hr.

The actual wander rates of the gyros were determined by measuring the amount of wander occurring about the platform roll, yaw, and pitch axes over a period of approximately ten minutes. This wander rate was composed of both constant wander and mass unbalance components.

A comparison between the actual wander rates and the wander rates obtained from measurements of current, is given in Table 5.

6 CONCLUSIONS

The accuracy with which gyro wander rate can be measured, depends largely on the uncertainty in the recorder and the torque motor calibration. The recorder errors arise mainly in estimating the mean value of the noise on the output trace.

In general, the experimental results showed good agreement between the recorded and actual gyro wander rates. In two or three cases the discrepancy was large - this is explained by the fact that during these runs, disturbances in the platform alignment loops were causing excessively large oscillations of torque motor current.

The maximum amount of low frequency noise which can be tolerated on the recorder output trace is about 1 deg/hr peak to peak. In order to reduce disturbances on the launcher, all calibrations should be done with the gantry surrounding the missile. Since the natural bending frequency of the missile is approximately 1 c.p.s., it is possible that noise components of this frequency will appear on the recorder trace. If necessary, the time constant of the recorder filter should be made as long as say 10 seconds.

It appears that gyro wander rates on the launcher can be measured with a standard deviation of uncertainty of between 0·02 deg/hr and 0·03 deg/hr.

LIST OF REFERENCES

Ref.No.	Author	Title, etc.
1	Draper C. S., Wrigley, W., Grohe, L. R.	The floating integrating gyro and its application to geometrical stabilization problems on moving bases. Aero Eng Review Vol.15, No.6, pp.46-62. June, 1956.
2	Carter, J. V.	T2502-3C gyro : Torquer linearity. R.A.E. Technical Memorandum No. IAP.701. January, 1960. Also see IAP Memo by K.J. Biggs to be published.
3	Watson, D. H.	Ground control servo loops for the Black Knight stabilized platform. G.W. Technical Note to be published.
4	Beltrame, G. E.	Feedback accelerometer characteristics with particular reference to the Kearfott single axis accelerometer. G.W. Technical Note to be published.
5	Topping J.	Errors of observation and their treatment. (Institute of Physics, Monographs for Students) pp.78-79.

ATTACHED: Appendix
Tables 1 to 5
Drgs. GW/P/9856 - 9857
Negs. 150,227 - 150,228
Detachable abstract cards

ADVANCE DISTRIBUTION:

Ministry of Aviation		R.A.E.	
Chief Scientist		Director	
CGWL		DD(E)	
DGBM		DD(A)	
GW(G & C)5	70 copies	RPE	
TIL	90 "	Radio	
		Structures	
		Arm	
		IAP	3 copies
		Instn	
		Trials	
		Aberporth	
		Patents	
		Bedford Library	
		Library	

APPENDIX 1

RELATION BETWEEN GYRO TORQUE MOTOR CURRENT AND GYRO WANDER RATE ON THE LAUNCHER

The bearing and elevation of the line of sight of the autotheodolite, and the vertical supplied by the levelling accelerometer, provide a set of reference axes from which the components of earth's rate appearing along the missile roll, yaw, and pitch axis directions can be deduced. These components of earth's rate would appear along the input axis of the respective gyros if no misalignment errors occurred. The components of earth's rate appearing along the missile roll yaw and pitch axis directions on the launcher will be:

Roll $\omega_R = 7.739$ deg/hr.

Yaw $\omega_r = 12.574$ deg/hr.

Pitch $\omega_p = 2.865$ deg/hr.

To obtain the gyro wander rates, the platform will be orientated in four positions at 90 deg about the pitch axis. The normal flight position, when the roll, yaw, and pitch gyro input axes coincide with the missile roll yaw and pitch axis directions respectively, is called the 0 deg position. The relevant equations for obtaining gyro wander rate are given below.

Let I_0, I_{90}, I_{180}, I_{270} = gyro torque motor current when the platform is orientated in the 0, 90, 180, 270 deg position respectively.

I_R, I_Y, I_P = gyro torque motor current required to overcome the earth's rate component which is assumed to appear along the gyro input axis when the input axis coincides with the missile roll, yaw and pitch axes respectively.

I_{CW} = torque motor current required to overcome ligament torques i.e. constant wander.

I_{MUIA} = torque motor current to overcome mass unbalance along the gyro input axis.

I_{MUSRA} = torque motor current to overcome mass unbalance along the gyro spin reference axis.

(i) Roll gyro torque motor current

The equations applying to the four orientations of the gyro are obtained by use of Fig.1.

$$I_0 = I_R + I_{MUSRA} + I_{CW}$$

In the 90 deg position the input axis points along the missile yaw axis direction and I_{MUSRA} is replaced by I_{MUIA}, because the effective mass unbalance is now that component which appears along the input axis.

$$I_{90} = I_Y + I_{MUIA} + I_{CW}$$

in the 180 deg position the earth's rate component seen by the gyro is the reverse of that seen in the 0 deg position and the mass unbalance torque is reversed.

$$\therefore \quad I_{180} = -I_R - I_{MUSRA} + I_{CW}$$

similarly

$$I_{270} = -I_Y - I_{MUIA} + I_{CW}$$

In the 180 deg and 270 deg positions the recorded torque motor currents will be of opposite sign to those of the 0 deg and 180 deg positions. Since the recorder input will be reversed it will be more convenient to use the modulus of the currents.

$$|I_{180}| = I_R + I_{MUSRA} - I_{CW}$$

$$|I_{270}| = I_Y + I_{MUIA} - I_{CW}$$

(ii) Yaw gyro torque motor current

These equations are similar to those for the roll gyro

$$|I_0| = I_Y + I_{CW}$$

$$|I_{90}| = I_R + I_{MUSRA} + I_{CW}$$

$$|I_{180}| = I_Y - I_{CW}$$

$$|I_{270}| = I_R + I_{MUSRA} - I_{CW}$$

(iii) Pitch gyro torque motor current

Here the sign of the current required to hold the platform in position does not change sign.

$$I_0 = I_P + I_{MUIA} + I_{CW}$$

$$I_{90} = I_P + I_{CW}$$

$$I_{180} = I_P - I_{MUIA} + I_{CW}$$

$$I_{270} = I_P + I_{CW}$$

TABLE 1

Equations for determining mass unbalance and constant wander terms from torque motor current

Gyro	Platform Positions	Quantity measured	Appropriate equations
Roll	0 deg and 180 deg	I_{CW}	$I_{CW} = \frac{\lvert I_0\rvert - \lvert I_{180}\rvert}{2}$
	0 deg and 180 deg	I_{MUSRA}	$I_{MUSRA} = \frac{\lvert I_0\rvert + \lvert I_{180}\rvert}{2} - I_R$
	90 deg and 270 deg	I_{CW}	$I_{CW} = \frac{\lvert I_{90}\rvert - \lvert I_{270}\rvert}{2}$
	90 deg and 270 deg	I_{MUIA}	$I_{MUIA} = \frac{\lvert I_{90}\rvert - \lvert I_{270}\rvert}{2} - I_Y$
Yaw	0 deg and 180 deg	I_{CW}	$I_{CW} = \frac{\lvert I_0\rvert - \lvert I_{180}\rvert}{2}$
	90 deg and 270 deg	I_{CW}	$I_{CW} = \frac{\lvert I_{90}\rvert - \lvert I_{270}\rvert}{2}$
	90 deg and 270 deg	I_{MUSRA}	$I_{MUSRA} = \frac{\lvert I_{90}\rvert + \lvert I_{270}\rvert}{2} - I_R$
Pitch	0 deg and 180 deg	I_{CW}	$I_{CW} = \frac{I_0 + I_{180}}{2} - I_P$
	0 deg and 180 deg	I_{MUIA}	$I_{MUIA} = \frac{I_0 - I_{180}}{2}$
	90 deg and 270 deg	I_{CW}	$I_{CW} = \frac{I_{90} + I_{270}}{2} - I_P$

TABLE 2

Standard deviation of misalignment and measuring errors on the launcher

Error Source	Type of error	Axis about which error appears	Standard deviation σ
3.1a	Misalignment of gyro input	Gyro output axis	2·9 arcmin
3.1b	axis notch on roll ring	Gyro spin reference axis	2·9 arcmin
3.2a	Gyro manufacturing	Gyro output axis	1·7 arcmin
3.2b	tolerance	Gyro spin reference axis	1.7 arcmin
3.3a	Misalignment of mirrors	Missile pitch axis	0·6 arcmin
3.3b		Missile roll axis	0·6 arcmin
3.4	Accelerometer alignment errors	Missile yaw axis	1.2 arcmin
	Mirror cross-coupling effect	Missile roll axis	0·3 arcmin
3.5a	Accelerometer axis errors	Missile yaw axis	0·4 arcmin
	Mirror cross-coupling effect	Missile roll axis	0·1 arcmin
3.5b	Accelerometer bias errors	Missile yaw axis	1·6 arcmin
	Mirror cross-coupling effect	Missile roll axis	0·4 arcmin
3.6a	Optical loop errors	Missile roll axis	0·1 arcmin
3.6b		Missile pitch axis	0·1 arcmin
3.6c	Accelerometer loop errors	Missile yaw axis	0.3 arcmin
	Mirror cross-coupling effect	Missile roll axis	0·1 arcmin
3.7	Recorder uncertainty	-	0·023°/hr
3.8	Uncertainty in torque motor scale factor	-	0·01%

TABLE 3

Overall uncertainty in obtaining mass unbalance and constant wander rates for the roll gyro in the 0 deg and 180 deg positions

Error source	Missile axis about which error appears	Standard deviation of error	Standard deviation of uncertainty at 0 deg	Standard deviation of uncertainty at 180 deg	Standard deviation of constant wander uncertainty	Standard deviation of mass unbalance uncertainty
3.1a	Pitch	2·9 arcmin	0·011°/hr	0·011°/hr	0°/hr	0·011°/hr
3.1b	Yaw	2·9 arcmin	0·002	0·002	0	0·002
3.2a	Pitch	1·7 arcmin	0·006	0·006	0	0·006
3·2b	Yaw	1·7 arcmin	0·001	0·001	0	0·001
3.3a	Pitch	0·6 arcmin	0	0·002	0·002	0·002
3.3b	Roll	0·6 arcmin	0	0	0	0
3.4	Yaw Roll	1.2 arcmin 0·3 arcmin	0·001	0·001	0	0·002
3.5a	Yaw Roll	0·4 arcmin 0·1 arcmin	0·0003	0·0003	0	0·0003
3.5b	Yaw Roll	1·6 arcmin 0·4 arcmin	0·0013	0·0013	0·0013	0
3.6	-	-	-	-	-	-
3.7	-	0·023°/hr	0·023	0·023	0·016	0·016
3.8	-	0·1%	0·008	0·008	0	0·008
Overall = $\sqrt{\Sigma\sigma^2}$ individual errors					0·016°/hr	0·022°/hr

TABLE 4

Overall uncertainty in measuring gyro wander rates

Gyro	Platform positions	Quantity measured	Standard deviation of uncertainty
Roll	0 deg and 180 deg	CW	0·016
	0 deg and 180 deg	MUSRA	0·022
	90 deg and 270 deg	CW	0·016
	90 deg and 270 deg	MUIA	0·022
Yaw	0 deg and 180 deg	CW	0·016
	90 deg and 270 deg	CW	0·016
	90 deg and 270 deg	MUSRA	0·022
Pitch	0 deg and 180 deg	CW	0·023
	0 deg and 180 deg	MUIA	0·016
	90 deg and 270 deg	CW	0·023

TABLE 5

Comparison of actual wander rates and wander rates obtained from current measurements

Run	Gyro 1		Gyro 2		Gyro 3	
No.	Actual wander rate	Measured wander rate	Actual wander rate	Measured wander rate	Actual wander rate	Measured wander rate
1	0·97°/hr	0·99°/hr	0·23°/hr	0·27°/hr	-0·05°/hr	-0·03°/hr
2	1·00	1·01	0·24	0·31	-0·03	0·00
3	1·01	1·03	0·23	0·26	-0·02	-0·01
4	1·02	1·04	0·22	0·25	-0·03	-0·01
5	0·97	1·01	0·26	0·23	-0·01	0·0
6	1·00	1·02	0·21	0·11	-0·02	+0·02
7	0·99	1·03	0·25	0·26	-0·03	-0·01
8	0·97	1·01	0·24	0·23	-0·04	-0·04
9	1·00	1·04	0·24	0·26	-0·03	+0·01
10	0·99	1·03	0·24	0·34	-0·03	-0·02
11	1·00	1·04	0·27	0·28	-0·01	-0·03

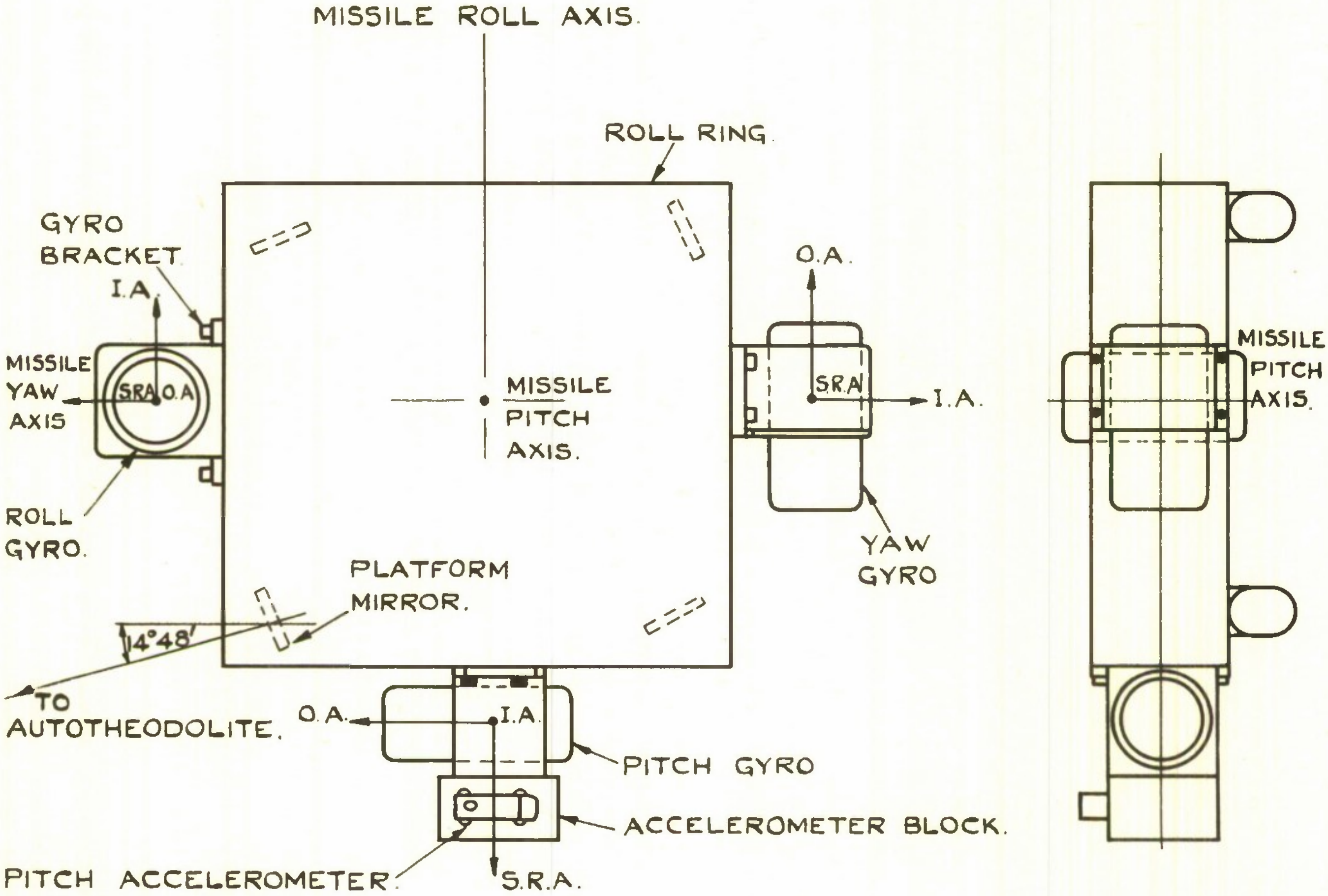

FIG. 1. SKETCH SHOWING ORIENTATION OF GYROS AND MIRRORS ON ROLL RING.

FIG. 2. KEARFOTT GYRO, SHOWING MOUNTING FLANGE & NOTCH

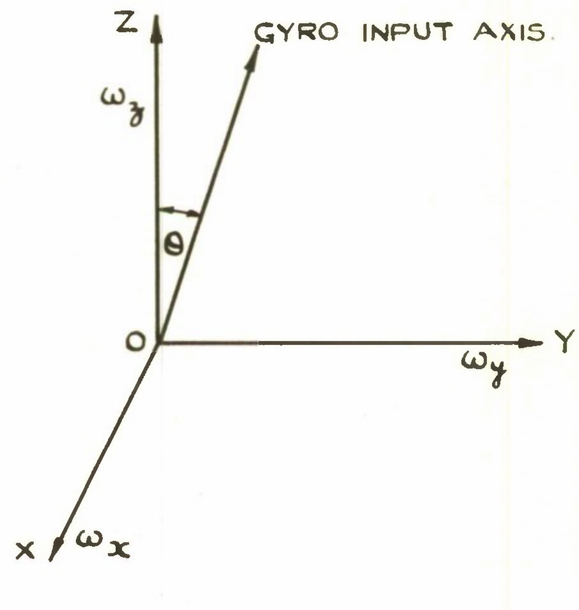

FIG. 3. EFFECT OF MISALIGNMENT OF GYRO INPUT AXIS FROM REFERENCE AXES.

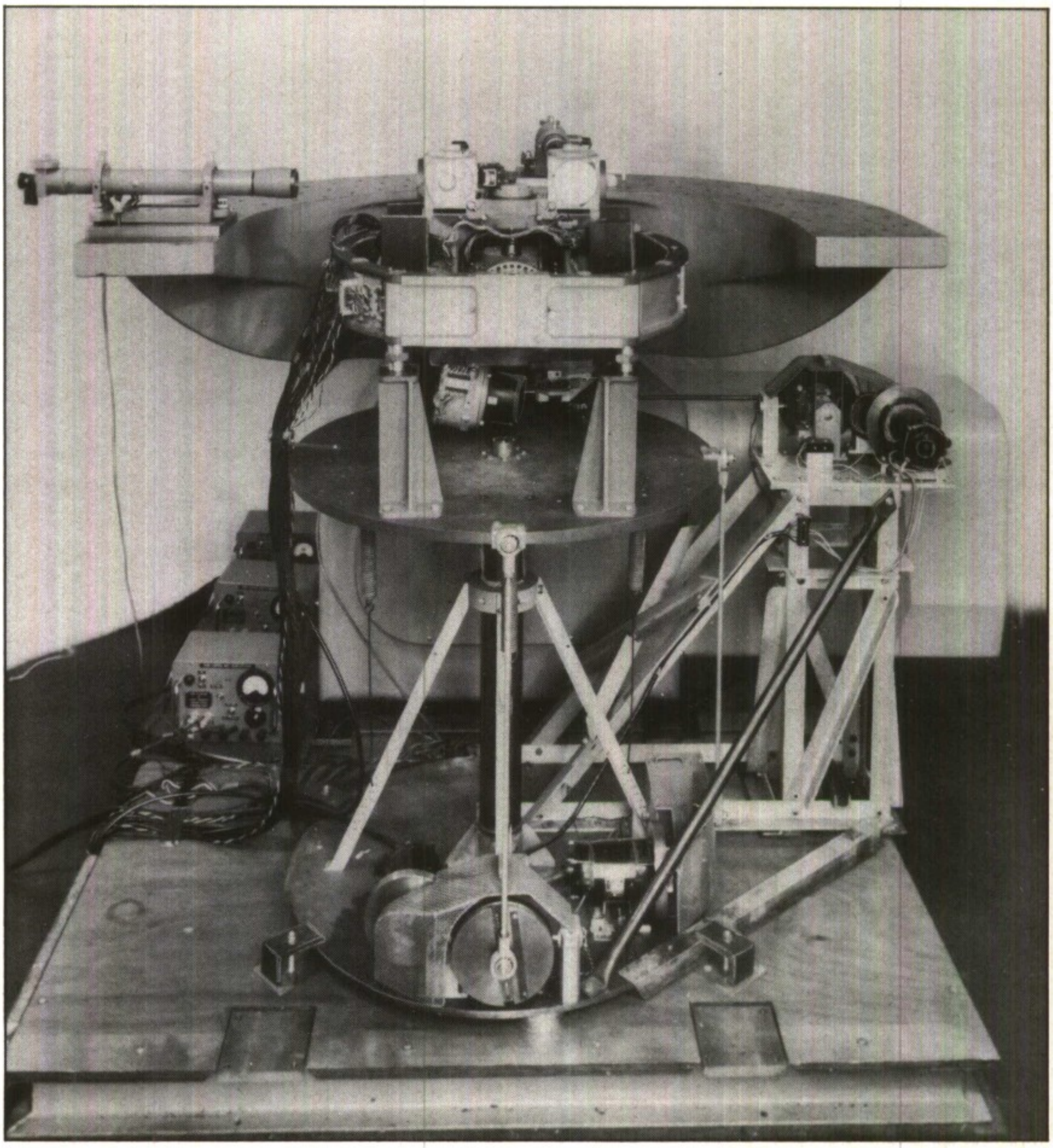

FIG. 4. EXPERIMENTAL PLATFORM & EQUATORIAL MOUNT

[dstl]

Information Centre
Knowledge Services
[dstl] *Porton Down*
Salisbury
Wilts
SP4 0JQ
22060-6218
Tel: 01980-613753
Fax 01980-613970

Defense Technical Information Center (DTIC)
8725 John J. Kingman Road, Suit 0944
Fort Belvoir, VA 22060-6218
U.S.A.

AD#: AD 323742

Date of Search: 11 December 2008

Record Summary: AVIA 6/21487
 Title: Assessment of the accuracy of measurement of gyro wander rate on the BLACK KNIGHT stabilised platform under pre-launch conditions
 Availability Open Document, Open Description, Normal Closure before FOI Act: 30 years
 Former reference (Department) TECHNICAL NOTES GW 564
 Held by The National Archives, Kew

This document is now available at the National Archives, Kew, Surrey, United Kingdom.

DTIC has checked the National Archives Catalogue website (http://www.nationalarchives.gov.uk) and found the document is available and releasable to the public.

Access to UK public records is governed by statute, namely the Public Records Act, 1958, and the Public Records Act, 1967.
The document has been released under the 30 year rule.
(The vast majority of records selected for permanent preservation are made available to the public when they are 30 years old. This is commonly referred to as the 30 year rule and was established by the Public Records Act of 1967).

This document may be treated as <u>UNLIMITED</u>.